Spring 2004

Physics 137B Section 1: Problem Set #2
Due: 5PM Friday Feb 6 in 2^{nd} floor LeConte-Birge Cross-Over

Suggested Reading for this Week:

- Bransden and Joachain (B& J) sections 6.8-6.10
- B& J section 8.1

Homework Problems:

1. *Adding two spins*

   In doing this problem, you may use the fact that the operators for the spin components of particle 1 commute with the operators for the spin components of particle 2.

   (a) Consider 2 electrons that are in an eigenstate of $(\vec{J})^2 = (\vec{S}_1 + \vec{S}_2)^2$ with eigenvalue $2\hbar^2$. Determine the value of $\vec{S}_1 \cdot \vec{S}_2$.

   (b) Now consider the specific eigenstate from part (a) that also has $\langle J_z \rangle = 0$. It can be written as

   $$|1,0\rangle = \frac{1}{\sqrt{2}} \left(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle \right)$$

   Calculate the expectation values of $\langle S_{1x} S_{2x} \rangle$, $\langle S_{1y} S_{2y} \rangle$ and $\langle S_{1z} S_{2z} \rangle$. Verify that their sum gives the correct result for $\left\langle \vec{S}_1 \cdot \vec{S}_2 \right\rangle$

2. *Precession in a Magnetic Field*

   A magnetic moment $\vec{\mu}$ is associated with a particle's spin by the expression $\vec{\mu} = \gamma \vec{S}$ where $\gamma \equiv g \frac{e}{2mc}$. Here g is called the gyromagnetic ratio and is an intrinsic property of the type of particle. For example $g = 2$ for an electron. In a magnetic field $\vec{B}$ the energy of orientation of the magnetic dipole is $-\vec{\mu} \cdot \vec{B}$, so the Hamiltonian H will contain a term $-\gamma(\vec{S} \cdot \vec{B})$. Using the general expression that relates the rate of change of the expectation value of an observable to the value of its commutator with H to show that $\vec{S}$ behaves dynamically like a classical top, that is:

$$\frac{\mathrm{d}}{\mathrm{dt}}\left\langle\vec{S}\right\rangle = \gamma\left\langle\vec{S}\right\rangle \times \vec{B}$$

3. *Matrix Form For Higher Spin States*

   Obtain the matrix representation of the angular momentum operatovs J_x, J_y and J_z for the case $j = \frac{3}{2}$

4. *Calculating Clebsh-Gordan Coefficients*

   A particle with spin $3/2$ is in a bound state with a particle of spin $1/2$. We define the total spin $\vec{S^{tot}}$ as the sum of the spins of these particles $\vec{S^{tot}} = \vec{S}_1 + \vec{S}_2$. Using the angular momentum raising and lowering operators, construct the states that are eigenstates of $(\vec{S^{tot}})^2$ and S_z^{tot}.

5. *Reading Clebsch-Gordan Tables*

   Anser the following questions using a Clebsh-Gordan table. A web pointer to such a table is available from our Web page.

   (a) Particle A is in the angular momentum state $\left|j = 1, m_j = 0\right\rangle$ and particle B is in the angular momentum state $\left|j = 1, m_j = -1\right\rangle$. Express the wave function for the system as a superposition of states of specific $(J^{tot})^2$ and J_z^{tot}. What is the probability that $j^{tot} = 2$?

   (b) A particle with spin $3/2$ is in an eigenstate of S_z with eigenvalue$\hbar/2$. This particle has an orbital angular momentum eigenvalue $\ell = 1$ and $m_\ell = 0$. Write down the wave function of this particle in the in the basis where the states are labled by the eigenstates of $(J^{tot})^2$ and J_z^{tot}. good quantum numbers.

6. *Non-Degenerate Perturbation Theory*

   A particle of mass m obeys the Hamiltonian

$$H = \frac{p^2}{2m} + \frac{1}{2}kx^2 + bx^4$$

   To first order in perturbation theory, find the energy and the wave function for the state with quantum number $n = 3$. Use the raising and lowering operators a and $a^\dagger$ to solve this problem.